# Highly Accurate Analysis of Magnetic Field by Local-Expansion Edge Element Method with Boundary Surface Integration

Takuya Uchiyama[1], Yuki Wakayama[1], Shinji Wakao[1],
Tadashi Tokumasu[2], Yasuhito Takahashi[3], and Koji Fujiwara[3]

[1]Waseda University, Tokyo 169-8555, Japan, wakao@waseda.jp
[2]Toshiba Corporation Infrastructure Systems and Solutions Company, Tokyo 183-8511, Japan
[3]Doshisha University, Kyoto 610-0321, Japan

**The Local-expansion Edge Element Method (LEEM) enables us to analyze magnetic field more accurately than the ordinary finite element method (FEM). This paper proposes the combination of LEEM and Boundary Surface Integration (BSI) as a post-processing for computational accuracy enhancement. In the proposed method, we also investigate an appropriate virtual boundary configuration for BSI in local-expansion region and achieve highly accurate analysis.**

***Index Terms*— local-expansion element, edge element, boundary surface integration, post-processing, magnetic field analysis.**

## I. Introduction

The Local-expansion Edge Element Method (LEEM), which is based on the local expansion theory known as the general solution of Laplace's equation, enables us to interpolate physical quantities in a target domain with higher order functions and perform highly accurate magnetic field analysis [1]. In this paper, we propose the combination of LEEM and Boundary Surface Integration (BSI) as a post-processing to make further enhancement of computational accuracy for magnetic shield problems, etc. By using BSI, we can accurately calculate magnetic field as smooth and continuous physical quantity. We also investigate the influence of virtual boundary configuration in local-expansion region on the computational accuracy.

## II. Formulation

### A. Local-expansion Edge Element Method

Fig. 1 shows the conceptual diagram of the proposed method. Fig. 2 shows the local-expansion element, e.g., a quadrangular pyramid, in which the vector potential $\boldsymbol{A}$ is expressed as the following equation by using $\boldsymbol{A}$ defined on each edge [1]-[4].

$$\boldsymbol{A} = \sum_{n=2}^{N}\left[\sum_{e=1}^{4}\phi_n(t)\{f_e(r,s)\nabla r + g_e(r,s)\nabla s\}A_{e\_n}^{rs} + \sum_{e=5}^{8}\frac{\mathrm{d}\phi_n(t)}{\mathrm{d}t}h_e(r,s)\nabla t\, A_{e\_n}^{t}\right], \quad (1)$$

$$\phi_n(t) = t^n \quad (n = 2,3,\cdots,N), \quad (2)$$

where $N$ is order of expansion, $f_e, g_e, h_e$ are scalar shape functions defined by TABLE I and $\phi_n$ is expansion function. The unknown variables ($A_{e\_n}^{rs}$, $A_{e\_n}^{t}$) are assigned on each edge of the LE, where the subscript $e\_n$ represents the edge number of $e$ with the expansion order of $n$. In this paper, instead of using (2), the orthogonalized expansion function is used to improve convergence characteristic of LEEM [1].

TABLE I
Expression Of $f_e, g_e$ And $h_e$

| $e$ | $f_e(r,s)$ | $g_e(r,s)$ | $e$ | $h_e(r,s)$ |
|---|---|---|---|---|
| 1 | (1- $s$)/4 | 0 | 5 | (1-$s$)(1-$r$)/4 |
| 2 | (1+$s$)/4 | 0 | 6 | (1-$s$)(1+$r$)/4 |
| 3 | 0 | (1-$r$)/4 | 7 | (1+$s$)(1+$r$)/4 |
| 4 | 0 | (1+$r$)/4 | 8 | (1+$s$)(1-$r$)/4 |

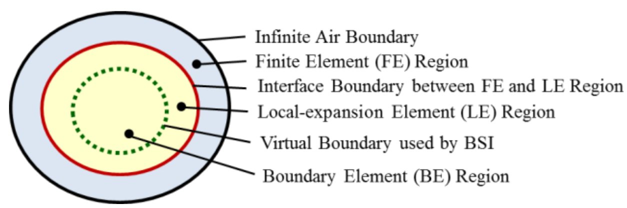


Fig. 1. Conceptual diagram of application of BSI to LEEM.

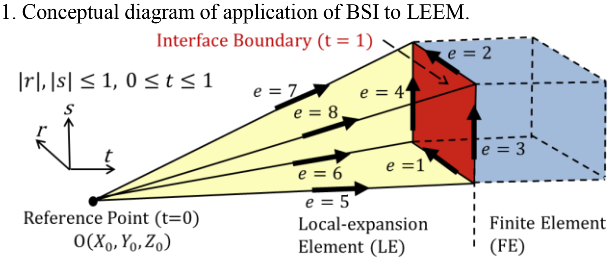


Fig. 2. Definition of local-expansion element.

### B. Application of Boundary Surface Integration in Post-processing

After carrying out the magnetic field analysis by LEEM, we set a virtual boundary surrounding the target domain inside the LE region as shown in Fig. 1. The boundary integral equation with magnetic scalar potential $\varphi$ is given as [5]

$$\frac{\Omega}{4\pi}\varphi(\boldsymbol{r}') = \frac{1}{4\pi}\int_{S}\left\{\frac{\boldsymbol{n}}{|\boldsymbol{r}-\boldsymbol{r}'|}\frac{\partial\varphi}{\partial n}(\boldsymbol{r}) - \frac{\boldsymbol{r}-\boldsymbol{r}'}{|\boldsymbol{r}-\boldsymbol{r}'|^3}\varphi(\boldsymbol{r})\right\}\cdot \mathrm{d}\boldsymbol{S}, \quad (3)$$

where $\Omega$ is the solid angle subtended by the target domain at the investigated point. $\partial\varphi/\partial n$ on the virtual boundary is directly given by LEEM. $\varphi$ on the virtual boundary can be derived by solving a linear system made by (3) with $\partial\varphi/\partial n$. Then, we can accurately obtain $\boldsymbol{B}$ at arbitrary points in the target domain as smooth and continuous physical quantity by calculating the negative gradient of (3).

## III. NUMERICAL EXAMPLES

We analyzed the permanent magnet model shown in Fig. 3. Magnetic flux densities were calculated on observation surfaces of which each one is divided into 100 × 100 lattice points, and were evaluated in terms of the accuracy by comparing with theoretical values [6].

Magnetic flux densities were calculated on the observation surface with varying $h$ from 1 to 30 by the ordinary FEM, LEEM, and the proposed method. Fig. 4 shows the averaged relative error in the cases of various virtual boundary sizes, i.e., interface boundary sizes $H$. On the whole, it seems that the proposed method is more effective than the ordinary FEM and LEEM from the viewpoint of computational accuracy.

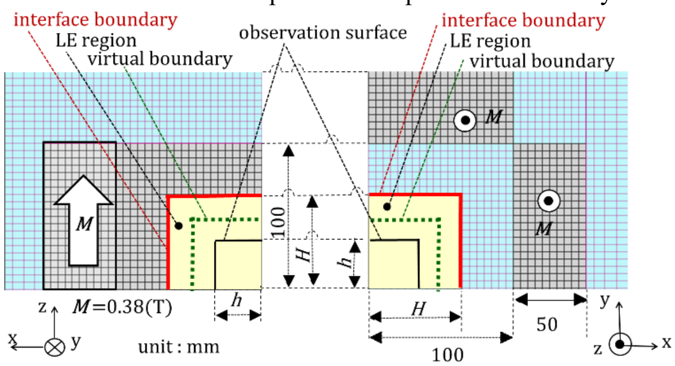


Fig. 3. One-eighth of permanent magnet model.

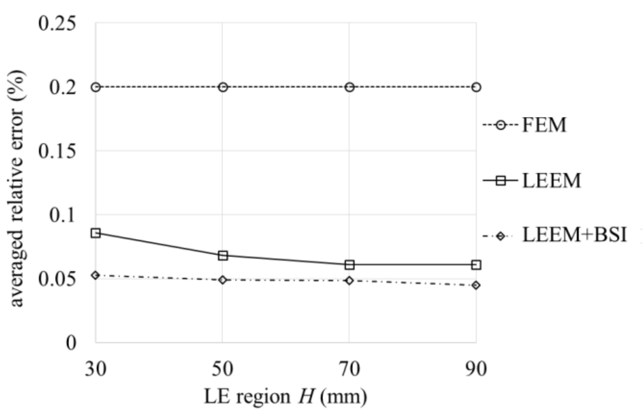


Fig. 4. Comparison of computational accuracy by FEM, LEEM, and proposed method.

## IV. IMPROVEMENT OF COMPUTATION ACCURACY BY USING SPHERICAL VIRTUAL BOUNDARY

We try further improvement of the computational accuracy by changing the shape of virtual boundary. A spherical virtual boundary in the LE region is set as shown in Fig. 5, which can reduce the deviation of absolute value of $\partial\varphi/\partial n$ on the virtual boundary.

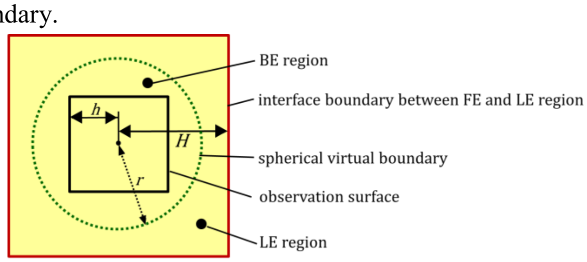


Fig. 5. Spherical virtual boundary set in local-expansion region.

Fig. 6 shows the averaged relative error on each observation surface calculated by the proposed method using the spherical virtual boundary or other methods. The results in Fig. 6 reveal that the proposed method using the spherical virtual boundary provides us the most accurate results.

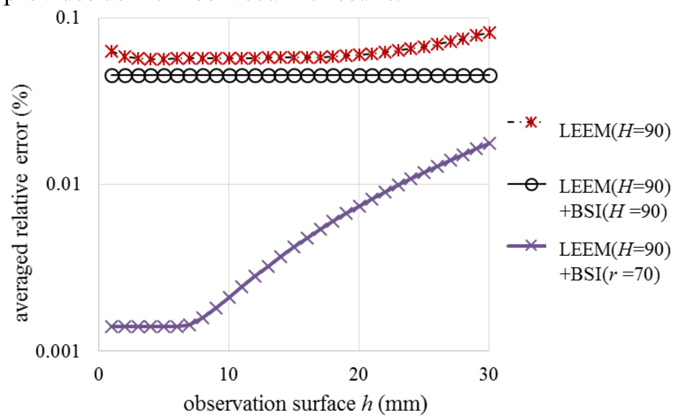


Fig. 6. Relative error of $|\boldsymbol{B}|$ on observation surface by proposed method using spherical virtual boundary or other methods.

## V. CONCLUSIONS

In this paper, we proposed the combination of local-expansion edge element method and boundary surface integration as a post-processing. The following is the summary of this paper.

(1) The combination LEEM and BSI as a post-processing enables us to analyze magnetic fields in the target domain with fairly high accuracy.

(2) It is effective to set the spherical virtual boundary in the LE region for further improvement of the computational accuracy.

In the full paper, more detailed formulation and numerical results demonstrating the effectiveness of the proposed method will be presented. We will also investigate an approximation method of the physical quantities on the virtual boundary for the further improvement of the computational accuracy.

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